

## LOGARITHME NÉPÉRIEN ET LIMITES

### Exercice

1. Soit  $f(x) = \frac{\cos(\pi x^2 - \frac{\pi}{3}) + \frac{1}{2}}{x-1}$  ; calculer  $\lim_{x \rightarrow 1} f(x)$ .

2.  $f(x) = \ln\left(\frac{ex+3}{x+5}\right)$  ; calculer  $\lim_{x \rightarrow +\infty} f(x)$ .

3.  $f(x) = \ln\left(\frac{x^2+3}{e^x}\right)$  ; calculer  $\lim_{x \rightarrow +\infty} f(x)$ .

4.  $\lim_{x \rightarrow +\infty} \frac{2 \ln x + 1}{2x}$ .

5.  $\lim_{x \rightarrow +\infty} x \ln\left(1 + \frac{1}{x}\right)$ .

### Correction

1.  $\lim_{x \rightarrow 1} \frac{\cos(\pi x^2 - \frac{\pi}{3}) + \frac{1}{2}}{x-1} = \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x-1} = f'(1)$  avec  $\begin{cases} f(x) = \cos(\pi x^2 - \frac{\pi}{3}) \\ f(1) = \cos(\pi - \frac{\pi}{3}) = \cos(\frac{2\pi}{3}) = -\frac{1}{2} \end{cases}$ .

On calcule donc  $f'(x) = -2\pi x \sin(\pi x^2 - \frac{\pi}{3})$  d'où  $f'(1) = -2\pi \sin(\pi - \frac{\pi}{3}) = -2\pi \sin \frac{2\pi}{3} = -\frac{2\pi\sqrt{3}}{2} = -\pi\sqrt{3}$ .

2.  $\lim_{x \rightarrow +\infty} \frac{ex+3}{x+5} = e \Rightarrow \lim_{x \rightarrow +\infty} \ln\left(\frac{ex+3}{x+5}\right) = \ln e = 1$ .

3.  $\lim_{x \rightarrow +\infty} \ln\left(\frac{x^2+3}{e^x}\right) = \lim_{x \rightarrow +\infty} [\ln(x^2+3) - \ln e^x] = \lim_{x \rightarrow +\infty} [\ln(x^2(1 + \frac{3}{x^2})) - x] = \lim_{x \rightarrow +\infty} [\ln x^2 + \ln\left(1 + \frac{3}{x^2}\right) - x]$ ,

or  $\lim_{x \rightarrow +\infty} \ln\left(1 + \frac{3}{x^2}\right) = \ln 1 = 0$  et  $\lim_{x \rightarrow +\infty} (\ln x^2 - x) = \lim_{x \rightarrow +\infty} (2 \ln x - x) = \lim_{x \rightarrow +\infty} x \left(2 \frac{\ln x}{x} - 1\right) = -\infty$  car

$\lim_{x \rightarrow +\infty} \frac{\ln x}{x} = 0$ .

4.  $\lim_{x \rightarrow +\infty} \frac{2 \ln x + 1}{2x} = \lim_{x \rightarrow +\infty} \frac{\ln x}{x} + \lim_{x \rightarrow +\infty} \frac{1}{2x} = 0$  car  $\lim_{x \rightarrow +\infty} \frac{\ln x}{x} = 0$  et  $\lim_{x \rightarrow +\infty} \frac{1}{2x} = 0$

5.  $\lim_{x \rightarrow +\infty} x \ln\left(1 + \frac{1}{x}\right) = \lim_{x \rightarrow +\infty} \frac{\ln\left(1 + \frac{1}{x}\right)}{\frac{1}{x}} = \lim_{X \rightarrow 0^+} \frac{\ln(1+X)}{X} = 1$ .